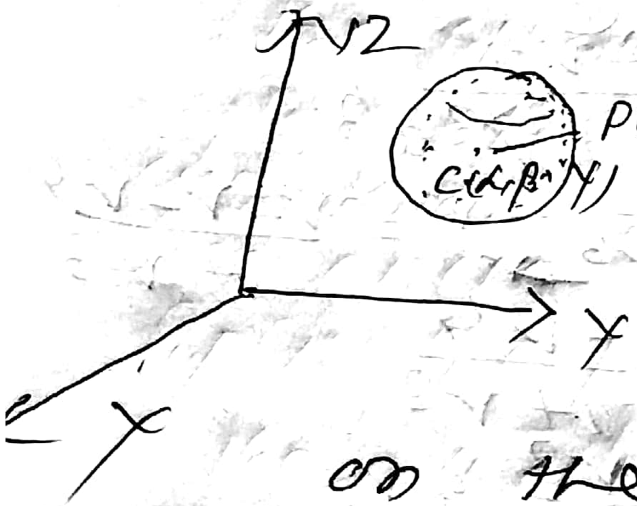
B.Sc. (Math) part I
paper - II
Topic:- The sphere (3D Geometry)

Sphere

Def:- The locus of point which moves such that its distance from a fixed point is always constant is called a sphere.

The fixed point is called the centre of sphere and the fixed distance is called the radius of the sphere.

Standard eqn. of a sphere

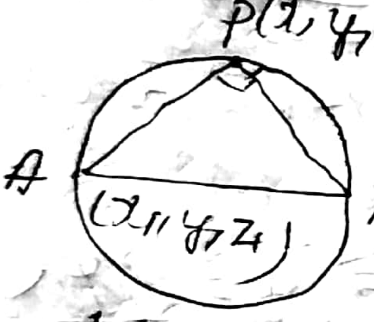


Let $C(\alpha, \beta, \gamma)$ be the centre of sphere and $P(x, y, z)$ any point on the sphere.

Let a be radius of sphere then $CP = a$

Now $CP^2 = (x-\alpha)^2 + (y-\beta)^2 + (z-\gamma)^2$ (distance formula) then $(x-\alpha)^2+(y-\beta)^2+(z-\gamma)^2=a^2$ is required eqn of sphere.

Theorem To find the equation of sphere described on the line segment joining two points (x_1,y_1,z_1) and (x_2,y_2,z_2) as diameter segment.



Let $A(x_1,y_1,z_1)$ and $B(x_2,y_2,z_2)$ be two given points and $P(x,y,z)$ be any point on the sphere. Let AB be diameter of the sphere.

Join AP and BP. Since we know that the angle in semi circle is right angle therefore AP is perpendicular to BP.

The direction ratios of AP and BP are
$x-x_1, y-y_1, z-z_1$ and $x-x_2, y-y_2, z-z_2$

$\because AP \perp BP$

$\therefore (x-x_1)(x-x_2)+(y-y_1)(y-y_2)+$

$(z-z_1)(z-z_2)=0$

This is required eqn. of Sphere.

Condition for Tangency

To find the ~~eqn~~ condition that the plane $ax+by+cz+d$ should touch the sphere $x^2+y^2+z^2+2ux+2vy+2wz+K=0$

Soln :- Let the equation of plane be

$$ax+by+cz+d=0 \quad \text{—— (1)}$$

and let the equation of the sphere be

$$x^2+y^2+z^2+2ux+2vy+2wz+K=0 \quad \text{—— (2)}$$

∴ Centre of the sphere

$$\equiv(-u,-v,-w)$$

and radius of sphere

$$=\sqrt{u^2+v^2+w^2-K^2}$$

If the plane (1) is a tangent plane on (2) then the lent of perp. from the centre to given plane must be equal to the radius of the sphere.

For this condition is given by

$$\frac{a(-u)+b(-v)+c(-w)+d}{\sqrt{a^2+b^2+c^2}} = \sqrt{u^2+v^2+w^2-K}$$

Squaring both side we get

$$\frac{(au+bv+cw-d)^2}{a^2+b^2+c^2} = u^2+v^2+w^2-K$$

or $(au+bv+cw-d)^2$

$$= (a^2+b^2+c^2)(u^2+v^2+w^2-K)$$

problem (1) Find the equation of sphere which passes through the point (α, β, γ) and the circle $x^2+y^2=a^2$ and $z=0$

Let the eqn. of the sphere passing through the given circle is

$$x^2+y^2+z^2-a^2+Kz=0 \quad (1)$$

If (1) passes through (α, β, γ) then $\alpha^2+\beta^2+\gamma^2-a^2+K\gamma=0$

$\alpha^2+\beta^2+\gamma^2-a^2$

Now, substituting the value K in ① we get

$$(x^2+y^2+z^2-a^2)-\frac{1}{\gamma}(\alpha^2+\beta^2+\gamma^2-a^2)z=0$$

$$\gamma(x^2+y^2+z^2-a^2)=(\alpha^2+\beta^2+\gamma^2-a^2)z$$

which is required equation of the sphere.

Problem ②: A plane passes through a fixed point (a,b,c) and cut the axes in A, B, C. Show that the locus of the centre of the sphere (O, ABC) is

$$\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=2$$

Soln: – Let the co-ordinate of points A, B, C are $(\alpha,0,0)$, $(0,\beta,0)$ and $(0,0,\gamma)$.

The eqn. of plane is

$$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=1$$

Since the plane is passes through the point (a,b,c) we have

$$\frac{a}{\alpha}+\frac{b}{\beta}+\frac{c}{\gamma}=1 \quad \text{—— ①}$$

The eqn. of the sphere

through O, A, B, C is

$x^2+y^2+z^2 - \alpha x - \beta y - \gamma z = 0$

the Co-ordinate (f, g, h) of the centre of sphere are

$f = \frac{\alpha}{2}, g = \frac{\beta}{2}, h = \frac{\gamma}{2}$

or $\alpha = 2f, \beta = 2g$ and $\gamma = 2h$

Substituting these values of f, g, h in (1) we get

$$\frac{a}{2f} + \frac{b}{2g} + \frac{c}{2h} = 1$$

or $\frac{a}{f} + \frac{b}{g} + \frac{c}{h} = 2$

Hence the locus of the centre (f, g, h) is

$$\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 2$$

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